

QCD Corrections to Radiative  $\Upsilon$  Decays

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We have calculated the next-to-leading order perturbative QCD corrections to the photon energy spectrum in radiative  $\Upsilon$  decays. The higher-order corrections significantly modify the shape of the spectrum, in particular at large photon energies, and thereby reduce the discrepancy between experimental data and previous leading-order predictions. The next-to-leading order calculation of the photon energy spectrum allows a more reliable determination of the strong coupling constant from radiative  $\Upsilon$  decays by restricting the analysis to the region of the energy spectrum that can be described by NLO perturbation theory.

**1. Introduction.** The calculations of heavy quarkonium decay rates are among the earliest applications of perturbative QCD [1]; they have been used to extract information on the QCD coupling at scales of the order of the heavy-quark mass. A consistent and rigorous framework for treating inclusive quarkonium decays has recently been developed, superseding the earlier non-relativistic potential model. The factorization approach [2] is based on the use of non-relativistic QCD (NRQCD) to separate the short-distance physics of heavy-quark annihilation from the long-distance physics of bound-state dynamics. The quarkonium decay rate can be expressed as a sum of terms, each of which factors into a short-distance coefficient and a long-distance matrix element. The short-distance coefficients are determined by the annihilation cross section for an on-shell  $Q\bar{Q}[n]$  pair in a colour, spin and angular-momentum configuration  $n$ , and can be calculated perturbatively in the strong coupling  $\alpha_s(m_Q)$ . The probability to find a  $Q\bar{Q}$  pair in the quarkonium at the same point and in the configuration  $n$  is parametrized by a non-perturbative matrix element  $\langle\mathcal{O}_n^\Upsilon\rangle$ , which is subject to the power counting rules of NRQCD and which can be evaluated numerically using NRQCD lattice simulations [3]. It is thus, in principle, possible to calculate the annihilation decay rates of heavy quarkonium from first principles, the only inputs being the heavy-quark mass  $m_Q$  and the QCD coupling  $\alpha_s(m_Q)$ . Fixing the heavy-quark mass from, for example, sum rule calculations for quarkonia, or considering ratios of different decay channels where the quark-mass dependence and the long-distance factor cancel, the analysis of quarkonium decay rates can provide information on the QCD coupling at the heavy-quark mass scale.

In this letter, we focus on the reaction  $\Upsilon \rightarrow \gamma + X$ , which is used for  $\alpha_s$  determinations from the quarkonium system and which provides detailed information on the decay dynamics through the analysis of the photon energy spectrum. We present the first calculation of the next-to-leading order perturbative QCD corrections to the photon spectrum. Besides being an interesting problem of perturbative QCD per se, the inclusion of higher-order corrections allows a NLO determination of the strong coupling constant, avoiding those regions of the energy spectrum that are affected by non-perturbative contributions.<sup>1</sup>

**2. The leading-order calculation.** At leading order in the non-relativistic velocity expansion in  $v^2$ , radiative  $\Upsilon$  decays proceed through the annihilation of a colour-singlet  $n = {}^3S_1^{(1)} b\bar{b}$  pair (in spectroscopic notation).<sup>2</sup> The photon energy

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<sup>1</sup>In the absence of a NLO prediction for the photon spectrum, model approaches have been used to extrapolate the experimental data into the region of low photon energies to obtain the total decay rate, introducing uncontrolled theoretical uncertainties. The model dependence of the corresponding NLO determination of  $\alpha_s$  can be avoided by restricting the analysis to the region of the energy spectrum that is described by NLO perturbation theory.

<sup>2</sup>At leading order in the velocity expansion, the NRQCD description of  $S$ -wave quarkonium decays is equivalent to the non-relativistic potential model, where the non-perturbative dynamics is

spectrum receives contributions from two different mechanisms. At leading order, one has

$$\frac{d\Gamma^{\text{LO}}(\Upsilon \rightarrow \gamma X)}{dx_\gamma} = \frac{d\Gamma_{\text{dir}}^{\text{LO}}}{dx_\gamma} + \int_{x_\gamma}^1 \frac{dz}{z} C_g(z) D_g^\gamma\left(\frac{x_\gamma}{z}\right), \quad (1)$$

where  $0 \leq x_\gamma = E_\gamma/m_b \leq 1$ . The first term on the right-hand side denotes the direct cross section, where the photon is radiated off a heavy quark, Fig. 1(a), while the second term represents the contribution from gluon fragmentation into a photon, Fig. 1(b). The fragmentation contribution, although of  $\mathcal{O}(\alpha\alpha_s^4)$  in the coupling

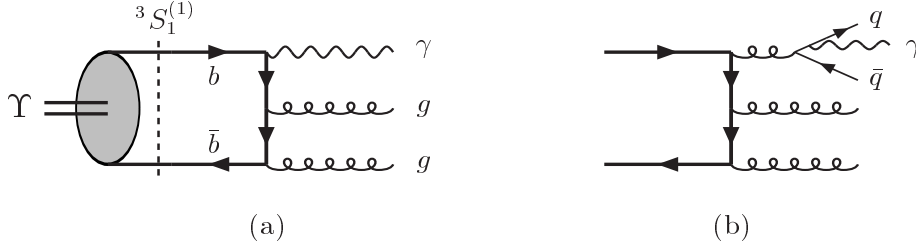


Figure 1: Generic leading-order Feynman diagrams for radiative  $\Upsilon$  decay: (a) direct contribution, (b) fragmentation contribution.

constant, is enhanced by a double logarithmic singularity  $\sim \ln^2(m_b^2/\Lambda^2) \sim 1/\alpha_s^2$  arising from the phase-space region where both the light-quark–antiquark splitting as well as the photon emission become collinear [4]. The fragmentation function  $D_g^\gamma$  is sensitive to non-perturbative effects and has to be extracted from experiment. A comparison between the direct component and the fragmentation contribution to the photon spectrum, performed at leading order, reveals that the fragmentation process is important in the low- $x_\gamma$  region, but suppressed with respect to the direct cross section for  $x_\gamma \gtrsim 0.3$  [4–5].<sup>3</sup> It is, however, important to point out that the total decay rate, including fragmentation processes,  $\Gamma(\Upsilon \rightarrow \gamma X) = \int_0^1 dx_\gamma d\Gamma/dx_\gamma$ , is not an infrared-safe observable. For  $x_\gamma \rightarrow 0$ , the fragmentation contribution rises like  $1/x_\gamma$ , characteristic of the soft bremsstrahlung spectrum, and cannot be integrated down to  $x_\gamma = 0$ . Therefore, in perturbation theory, only the photon energy spectrum can be calculated for  $x_\gamma \neq 0$ , after collinear singularities have been factorized into the fragmentation functions.

In the following, we will focus on the direct contribution of the energy spectrum, which dominates in the region  $x_\gamma \gtrsim 0.3$  where accurate experimental data are described by a single long-distance factor related to the bound-state’s wave function at the origin. In the case of  $P$ -wave quarkonia, the potential-model calculations at  $\mathcal{O}(\alpha_s^3)$  encounter infrared divergences, which cannot be factored into a single non-perturbative quantity. Within the NRQCD approach, this problem finds its natural solution since the infrared singularities are factored into a long-distance matrix element related to higher Fock-state components of the quarkonium wave function.

<sup>3</sup>At next-to-leading order, the fragmentation contribution involves also quark fragmentation, which is much harder than gluon fragmentation and may be important at larger values of  $x_\gamma$ .

available. At leading order, one finds [6]

$$\frac{1}{\Gamma_{\text{dir}}^{\text{LO}}} \frac{d\Gamma_{\text{dir}}^{\text{LO}}(\Upsilon \rightarrow \gamma X)}{dx_\gamma} = \frac{2}{(\pi^2 - 9)} \left( 2 \frac{1 - x_\gamma}{x_\gamma^2} \ln(1 - x_\gamma) - 2 \frac{(1 - x_\gamma)^2}{(2 - x_\gamma)^3} \ln(1 - x_\gamma) \right. \\ \left. + \frac{2 - x_\gamma}{x_\gamma} + \frac{x_\gamma(1 - x_\gamma)}{(2 - x_\gamma)^2} \right). \quad (2)$$

To very good accuracy, Eq. (2) can be approximated by a linear spectrum:  $1/\Gamma_{\text{dir}}^{\text{LO}} d\Gamma_{\text{dir}}^{\text{LO}}/dx_\gamma \approx 2x_\gamma$ . The corresponding leading-order decay rate is given by [7]

$$\Gamma_{\text{dir}}^{\text{LO}}(\Upsilon \rightarrow \gamma X) = \frac{16}{27} \alpha \alpha_s^2 e_b^2 (\pi^2 - 9) \frac{\langle \mathcal{O}_1^\Upsilon(^3S_1) \rangle}{m_b^2}, \quad (3)$$

where  $e_b$  is the charge of the  $b$  quark. Up to corrections of  $\mathcal{O}(v^4)$ , the non-perturbative NRQCD matrix element is related to the  $\Upsilon$  wave function at the origin through  $\langle \mathcal{O}_1^\Upsilon(^3S_1) \rangle \approx 2N_C |\varphi(0)|^2$ , with  $N_C = 3$  the number of colours.

The linear rise with  $x_\gamma$  predicted by LO perturbative QCD, Eq. (2), is not supported by the experimental data [8]. In particular for  $x_\gamma \gtrsim 0.7$  the discrepancy is significant. The leading-order estimate of the photon energy spectrum demands several theoretical refinements: the inclusion of (i) higher-order perturbative QCD corrections; (ii) relativistic corrections due to the motion of the  $b$  quarks in the  $\Upsilon$  bound state; and (iii) hadronization effects that may become important close to the phase-space boundary. The relativistic corrections to the photon energy spectrum, including colour-octet contributions, have been analysed in [9, 5]. Although potentially sizeable, they are concentrated in the upper and lower end-point region and do not significantly modify the shape of the spectrum in the intermediate energy range  $0.4 \lesssim x_\gamma \lesssim 0.9$ . Hadronization effects have been studied in the context of different models [10] and may become important when  $x_\gamma \rightarrow 1$ . However, these non-perturbative contributions are not expected to modify the energy distribution significantly in the intermediate region  $x_\gamma \sim 0.7$ . The calculation of the next-to-leading order perturbative QCD corrections to the photon spectrum is the subject of this letter.

**3. Next-to-leading order QCD corrections.** Generic diagrams that build up the decay rate in  $\mathcal{O}(\alpha\alpha_s^3)$  are drawn in Fig. 2. The evaluation of the next-to-leading order corrections involves two parts, the virtual corrections, generated by virtual particle exchange, and the real corrections, which originate from real gluon radiation as well as from gluon splitting into light-quark–antiquark pairs [11].

The virtual amplitude consist of 105 diagrams, including self-energy and vertex corrections for photon and gluons (2a,b) as well as a large number of box diagrams (2c–e). The evaluation of these amplitudes has been performed in the Feynman gauge. The ultraviolet and infrared singularities have been extracted using dimen-

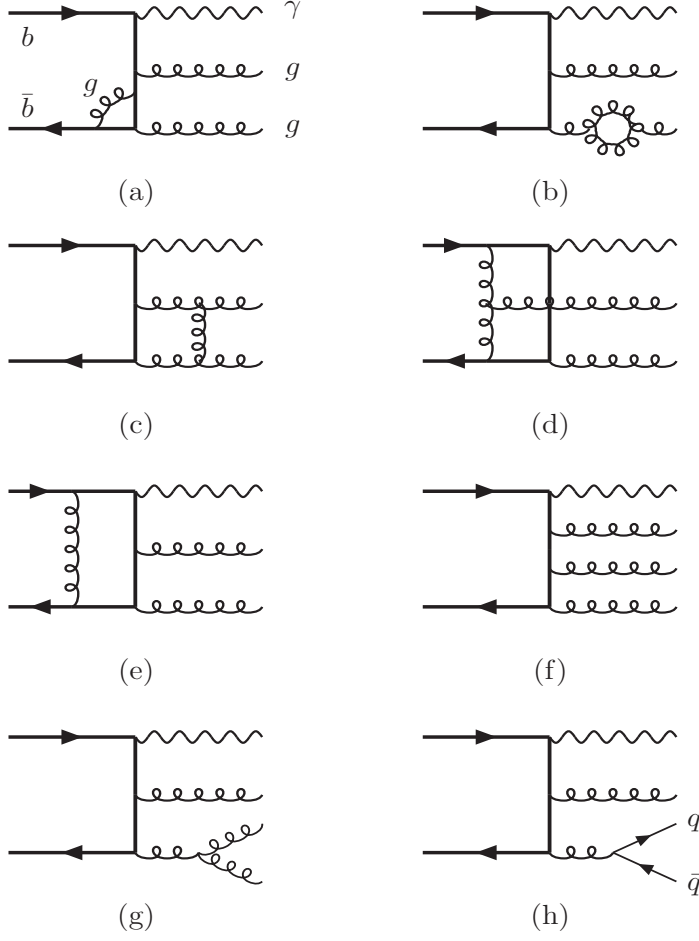


Figure 2: Generic next-to-leading order Feynman diagrams for direct radiative  $\Upsilon$  decay.

sional regularization, leading to poles in  $\epsilon = (n - 4)/2$ . The Feynman integrals containing loop momenta in the numerator have been reduced to a set of scalar integrals using an adapted version of the reduction program outlined in [12]. This program has been extended to treat  $n$ -dimensional tensor integrals with linear-dependent propagators to account for the special kinematical situation in the non-relativistic approximation to  $\Upsilon$  decays. Using the Feynman parameter technique, we have obtained analytical results for the scalar integrals; these will be presented in a forthcoming publication. The masses of the  $n_{\text{lf}} = 4$  light quarks have been neglected, while the mass parameter of the  $b$  quark has been defined on-shell. We have carried out the renormalization of the QCD coupling in the  $\overline{\text{MS}}$  scheme, including  $n_{\text{lf}} = 4$  light flavours in the corresponding  $\beta$  function. The bottom quark has been decoupled from the evolution of  $\alpha_s$  by subtracting its contribution at vanishing momentum transfer [13]. The exchange of Coulombic gluons in diagram (2e) leads to a singularity  $\sim \pi^2/2\beta_R$ , which can be isolated by introducing a small relative quark velocity  $\beta_R$ . The Coulomb-singular part can be associated with the interquark potential of the bound state and has to be factored into the non-perturbative NRQCD matrix

element. Only the exchange of transverse gluons contributes to the next-to-leading order expression for the short-distance annihilation rate.

The real corrections are generated by gluon radiation off quarks and gluons (2f,g) and by gluon splitting into light-quark–antiquark pairs (2h). The 54 diagrams have been calculated in the Feynman gauge. For the sum of the gluon polarization we have used  $\sum \varepsilon_\mu \varepsilon_\nu = -g_{\mu\nu}$  and the unphysical longitudinal gluon polarizations have been removed by adding ghost contributions. The real emission cross section contains singularities in the limit where the gluon splitting into gluon or light-quark–antiquark pairs becomes soft or collinear. We have used the subtraction method [14] as formulated in [15] to extract the singular parts of the real cross section in  $n$  dimensions. The method of [15] is based on the fact that the soft and collinear limits of a real emission matrix element can be expressed in terms of a convolution of process-independent splitting functions and the leading-order matrix element. Subtracting from the real matrix element counterterms constructed accordingly, the infrared and collinear singularities cancel and the phase-space integration can be performed numerically in four dimensions. The counterterms can be integrated analytically in  $n$  dimensions over the phase space of the extra emitted parton, leading to poles in  $\epsilon = (n - 4)/2$ . When the integral of the subtraction counterterm and the virtual cross section are combined, the infrared and collinear singularities cancel. The analytical result for the matrix element squared has been implemented in a Monte Carlo integration program, so that not only the inclusive decay rate but any distribution can be generated at next-to-leading order.

As mentioned above, owing to the presence of fragmentation processes at small photon energies, the total decay rate is not an infrared-safe observable. We will nevertheless present a next-to-leading order result for the perturbatively calculable direct contribution to the total decay rate, in order to compare our calculation with a previous NLO estimate of that quantity. We find

$$\Gamma_{\text{dir}}^{\text{NLO}}(\Upsilon \rightarrow \gamma X) = \Gamma_{\text{dir}}^{\text{LO}} \left[ 1 - \frac{\alpha_s(\mu)}{\pi} \left( 1.71 + \beta_0(n_{\text{lf}}) \ln \frac{2m_b}{\mu} \right) \right], \quad (4)$$

where  $\beta_0(n_{\text{lf}}) = (11N_C - 2n_{\text{lf}})/3$ . The theoretical uncertainty due to the numerical phase-space integration has been reduced to  $\lesssim 0.5\%$ . The new, more accurate result for the total rate is consistent with the previous calculation [16], which yielded  $\Gamma_{\text{dir}}^{\text{NLO}}/\Gamma_{\text{dir}}^{\text{LO}} = (1 - \alpha_s(1.67 \pm 0.36)/\pi)$  for  $\mu = 2m_b$ , with a considerable theoretical uncertainty of  $\sim \pm 20\%$  coming from the numerical evaluation of the loop integrals.<sup>4</sup>

The central part of this work is the calculation of the photon energy spectrum at next-to-leading order. The result is presented in Fig. 3. The QCD corrections significantly flatten and deplete the spectrum for  $x_\gamma \gtrsim 0.75$ . The shape of the NLO spectrum is in qualitative agreement with the experimental data and indicates

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<sup>4</sup>A prescription for fixing the renormalization scale  $\mu$  has been given in [17]. The analysis implies that it is advantageous to consider ratios like  $\Gamma(\Upsilon \rightarrow \gamma gg)/\Gamma(\Upsilon \rightarrow ggg)$  for  $\alpha_s$  extractions.

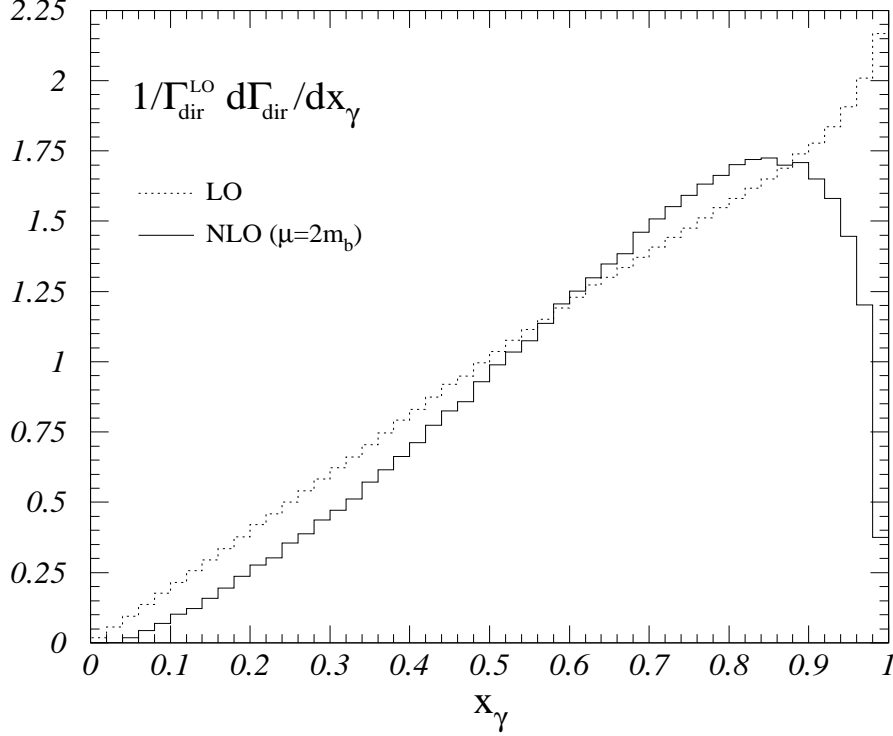


Figure 3: Photon energy spectrum for direct radiative  $\Upsilon$  decays in leading and next-to-leading order ( $\alpha_s = 0.2$ ).

that the discrepancy between theory and experiment at large photon energies can be reduced by the inclusion of higher-order QCD corrections. A proper comparison with the recent measurements requires the inclusion of experimental aspects, such as energy resolution and efficiency, and will be presented in a forthcoming publication.

Finally, let us briefly comment on the general structure of the higher-order corrections near the upper end-point of the photon energy spectrum. It has been argued that potentially large logarithms  $\ln(1 - x_\gamma)$  associated with the imperfect cancellation between real and virtual emission of soft gluons as  $x_\gamma \rightarrow 1$  may contribute to all orders in perturbation theory [18]. The resummation of these soft-gluon effects may then give rise to a Sudakov suppression  $\sim 1/\exp(\alpha_s \ln^2(1 - x_\gamma))$  near the end-point of the photon energy spectrum. In contrast to the previous claim [18], a recent analysis [19] finds that the logarithms of  $(1 - x_\gamma)$  cancel at each order in the perturbative expansion, as long as the  $\Upsilon$  decay proceeds through the annihilation of a colour-singlet  $b\bar{b}$  pair. While the numerical NLO calculation of the spectrum as presented in Fig. 3 is not suited to definitely resolving that issue, it may be possible to derive, from the full NLO result, a simple analytic expression of the NLO cross section near the end-point region. We leave such a calculation for a forthcoming publication.

In conclusion, the photon spectrum in radiative  $\Upsilon$  decays is a very interesting laboratory to study the effects of higher-order perturbative QCD corrections. The next-to-leading order calculation presented here shows that the intermediate region of the energy spectrum can be adequately accounted for by perturbative QCD. Fragmentation processes, relativistic corrections and hadronization effects have to be included to further improve the theoretical description of the spectrum in the end-point regions. In this respect, the NLO calculation is vital to disentangle perturbative effects from non-perturbative physics and relativistic corrections, and thus test the hadronization models that have been suggested in the literature. Finally, a more reliable NLO extraction of  $\alpha_s$  from radiative  $\Upsilon$  decays is possible by restricting the analysis to the perturbatively calculable part of the energy spectrum.

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